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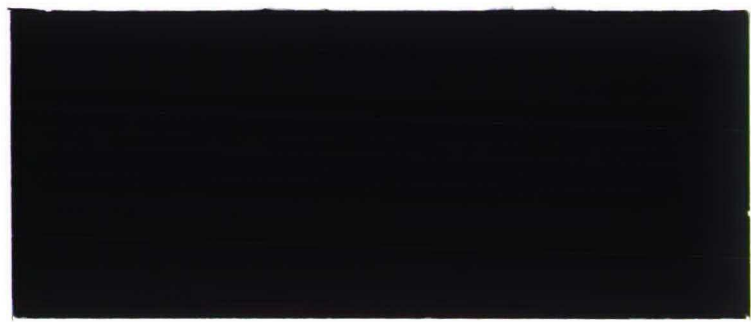
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DEPARTMENT OF ECONOMICS
RESEARCH MEMORANDUM



**GRAPHS COSPECTRAL WITH
DISTANCE-REGULAR GRAPHS**

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E. Spence

FEW 623

Communicated by Prof.dr. M.H.C. Paardekooper

Graphs Cospectral with Distance-Regular Graphs

W.H. Haemers and E. Spence

Abstract

We determine all graphs with the spectrum of a distance-regular graph with at most 30 vertices (except possibly for the Taylor graph on 28 vertices).

1. Introduction

The spectrum of a graph (i.e. the multiset of eigenvalues of its $(0, 1)$ adjacency matrix) gives much information about the structure of the graph. It indicates for instance if a graph is complete, regular, connected and regular, strongly regular, or bipartite. For a regular graph, the spectrum also determines the girth g as well as the number of g -gons through each vertex. See [4]. In general, however, it cannot be seen from the spectrum that a graph is distance-regular with diameter d when $d \geq 3$ (a distance-regular graph with $d \leq 2$ is complete or strongly regular). Counter-examples are given by Hoffman [8] for $d \geq 4$ (see also [1]) and by the first author [7] for $d = 3$. On the other hand, in some cases distance-regularity is determined by the spectrum even for $d \geq 3$. The present paper gives an (almost) precise state of these affairs for all spectra that are feasible for a distance-regular graph on n vertices for $n \leq 30$. For a spectrum Σ , let us denote by gr_{Σ} the number of graphs with this spectrum and by dr_{Σ} the number of distance-regular graphs with spectrum Σ . Clearly $gr_{\Sigma} \geq dr_{\Sigma}$ and equality means that distance regularity is recognizable from Σ . It is an easy and well-known result that $gr_{\Sigma} = dr_{\Sigma} = 1$ if Σ is the spectrum of an n -gon or a complete bipartite graph minus a complete matching. For all other feasible spectra for distance-regular graphs with $d \geq 3$ and $n \leq 30$ the values of dr_{Σ} and gr_{Σ} are exhibited in the table of Appendix 1.

As general references we use Brouwer Cohen and Neumaier [1] for distance-regular graphs and Cvetković, Doob and Sachs [4] for spectra of graphs. Both subject are also treated in the recent book of Godsil [5]. For the following four lemmas we refer to [7].

Lemma 1.1 *If Γ is cospectral with a distance-regular graph Δ with one of the properties below, then Γ is distance-regular.*

Δ has diameter 3 and $\mu = 1$,

Δ is bipartite with diameter 3, or

Δ has diameter d and girth $g \geq 2d - 1$.

The parameter $\mu (= c_2)$ of Δ gives the number of common neighbours of two vertices at distance 2. A bipartite distance-regular graph with $d = 3$ is the incidence graph of a symmetric $2-(v, k, \lambda)$ design (abbreviated to: $IG(v, k, \lambda)$).

Lemma 1.2 *Let Γ be a graph cospectral with a distance-regular graph with diameter 3 and k_2 vertices at distance 2 from each vertex.*

- i. *Any vertex of Γ has at least k_2 vertices at distance 2.*
- ii. *If a vertex γ of Γ has k_2 vertices at distance 2, then Γ is distance-regular around γ .*
- iii. *If every vertex of Γ has k_2 vertices at distance 2, then Γ is distance-regular.*

Lemma 1.3 *Let Γ be a graph cospectral with a bipartite distance-regular graph with diameter d and k_i vertices at distance i from each vertex ($i = 0, \dots, d$). If Γ has also k_i vertices at distance i from each vertex for $i = 0, \dots, d$, then Γ is distance-regular.*

Let Γ be a graph with a partition of the vertices into two parts V_1 and V_2 say. Consider the following operation. Delete each edge of Γ between V_1 and V_2 , and insert an edge between V_1 and V_2 for each nonadjacent pair of vertices from Γ . (Adjacency within V_1 and V_2 is left unchanged.) This operation is called Seidel switching with respect to the given partition (see Seidel [11]). Two graphs are (Seidel) switching equivalent if one can be obtained from the other by Seidel switching with respect to some partition. The Seidel spectrum of Γ is the spectrum of the ∓ 1 adjacency matrix $J - 2A + I$, where A is the $(0,1)$ -adjacency matrix of Γ (J denotes the all-one matrix and I the identity matrix). Switching equivalent graphs have the same Seidel spectrum. Also for the $(0,1)$ -adjacency matrix there is a switching operation that leaves the spectrum invariant.

Lemma 1.4 *Let A be a symmetric $(0,1)$ -matrix, partitioned as follows:*

$$A = \begin{bmatrix} A_{1,1} & \cdots & A_{1,\ell} & S_{1,1} & \cdots & S_{1,m} \\ \vdots & & \vdots & \vdots & & \vdots \\ A_{1,\ell}^T & \cdots & A_{\ell,\ell} & S_{\ell,1} & \cdots & S_{\ell,m} \\ S_{1,1}^T & \cdots & S_{\ell,1}^T & B_{1,1} & \cdots & B_{1,m} \\ \vdots & & \vdots & \vdots & & \vdots \\ S_{1,m}^T & \cdots & S_{\ell,m}^T & B_{1,m}^T & \cdots & B_{m,m} \end{bmatrix},$$

such that each block has constant row (and column) sum. Suppose that for each block $S_{i,j}$ all, none, or half of the entries are equal to 1. Then the matrix obtained from A by replacing each half-filled block $S_{i,j}$ by its complement is cospectral with A .

The switching of this lemma provides an important (but not the only) tool for the construction of graphs cospectral with a given graph. The concept is due to Godsil and McKay [6] (see also Godsil [5]) and is a special case of Seidel switching if all matrices $S_{i,j}$ are half filled.

2. Validation; theoretic part

In this section we will give an account for those results in the table of Appendix 1 that did not need a computer. The list of feasible spectra up to 30 vertices was kindly generated by A.E. Brouwer. The definition of feasible we use is the one given in Brouwer, Cohen and Neumaier [1] p.133. The same book gives the values of dr_Σ , except for case 21 and 22. For these cases we need to know that there exist precisely five symmetric 2-(15, 7, 3) designs, two of which are each others dual; see Nandi [9]. Therefore there are four non-isomorphic incidence graphs of such designs, and (of course) the same is true for the complementary designs.

For the cases 2, 3, 4, 10, 11, 12, 15, 16, 19, 21, 22 and 23 we have $dr_\Sigma = gr_\Sigma$ by Lemma 1.1. The cases 1, 6, and 9 are taken care of by the following proposition.

Proposition 2.1 *The Icosahedron graph, the Pappus graph and the Dodecahedron graph are characterized by their spectra.*

Proof. It suffices to prove that graphs with the given spectra are distance-regular. Suppose Γ has an adjacency matrix A with spectrum

$$\Sigma = \{5^1, \sqrt{5}^3, -1^5, -\sqrt{5}^3\}$$

which is the spectrum of the icosahedron graph. Then $C = A^2 + (1 - \sqrt{5})A - \sqrt{5}I$ is positive semi-definite. Suppose also that two non-adjacent vertices have μ common neighbours. Then C has a principal submatrix

$$\begin{bmatrix} 5 - \sqrt{5} & \mu \\ \mu & 5 - \sqrt{5} \end{bmatrix},$$

and hence $5 - \sqrt{5} - \mu \geq 0$, so that $\mu \leq 2$. For a vertex γ of Γ let Δ be the subgraph of Γ induced by the vertices at distance at least 2 from γ . Then we easily have that Δ has 6 vertices and 10 edges. Moreover $\mu \leq 2$ implies that Δ has no vertex of degree smaller than 3. Therefore the complement of Δ is one of the following four graphs: P_6 (P_n is the path with n vertices), $K_3 + P_2$, $K_2 + C_4$ (C_n denotes the n -gon), or C_5 with an isolated vertex. For the first three cases there are non-adjacent vertices in Δ with at least three common neighbours, which contradicts $\mu \leq 2$. Hence γ has 5 vertices at distance 2 and one at distance 3. So Γ is distance regular by Lemma 1.2 (or by straightfoward verification).

Next suppose that Γ has the spectrum of the Pappus graph. Since the Pappus graph is

3-regular and bipartite of girth 6, so is Γ . Hence each vertex of Γ has 6 vertices at distance 2 and therefore 6 vertices at distance 3 (since the diameter cannot exceed 4) and 2 vertices at distance 4. Now Lemma 1.3 gives that Γ is distance regular.

Finally we consider a graph Γ cospectral with the dodecahedron graph. Then Γ is 3-regular, has diameter at most 5, has girth 5 and has three pentagons through each vertex. The total number of closed walks of length 6 in Γ equals $\text{trace}(A^6)$, which can be derived from the spectrum. This implies that Γ has no hexagons, because the dodecahedron graph has no hexagons and because both graphs have the same number of trivial closed walks of length 6. So two pentagons have at most one edge in common and it follows that any two intersecting edges lie in a unique pentagon. Now it is straightforward to verify that there is a unique graph with all the mentioned properties. \square

The value of gr_Σ is due to Hoffman [8] for case 5 and due to Bussemaker and Cvetković [2] for case 8. For the remaining cases (7, 13, 14, 17, 18 and 20) we determined the values of gr_Σ by a computer search. But just to observe that $gr_\Sigma > dr_\Sigma$ there is an easy and computer free argument for most cases, which we give below. For the tetrahedral graph, the argument can be found in [7], but it seemed appropriate to give it here as well. The tetrahedral graph $J(v, 3)$ has as vertex set all 3-subsets of a v -set, where two subsets are adjacent whenever the intersection has size 2.

Proposition 2.2 *For $v \geq 6$ there exists a graph cospectral with the tetrahedral graph $J(v, 3)$, but not distance-regular.*

Proof. Fix a 4-subset X of the v -set. Partition the 3-subsets according the intersection sizes with X . This partition satisfies the hypotheses of Lemma 1.4 ($\ell = 1$ and $A_{1,1} = J - I$ corresponds to the four 3-subsets of X). It is easily checked that the graph obtained after switching is not distance-regular. \square

The Desargues graph is the bipartite double of the Petersen graph. This means that it has an adjacency matrix

$$\begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix},$$

where A is the adjacency matrix of the Petersen graph.

Proposition 2.3 *There exists a graph cospectral with the Desargues graph, but not distance-regular.*

Proof. Partition the Petersen graph into a 4-coclique and the six remaining vertices. This gives a partition of the vertex set of the Desargues graph into four parts to which Lemma 1.4 applies (with $\ell = 1, m = 3$). It is easily checked that, after switching, two vertices that correspond to the same Petersen-vertex have 3 common neighbours. Therefore

the graph is not distance-regular. □

See [2] for a picture of the graphs of the above proposition.

Hadamard graphs are distance-regular graphs with $n = 8\mu$ vertices, degree $k = 2\mu$ and the following spectrum and intersection array:

$$\{k^1, \sqrt{k}^k, 0^{2k-1}, -\sqrt{k}^k, -k^1\}, \quad \{k, k-1, \mu, 1; 1, \mu, k-1, k\},$$

respectively. They exist if and only if a Hadamard matrix of order k exists. Thus Hadamard graphs cannot exist if μ is odd and greater than 1. The intersection array, however, is feasible for all μ , and in fact graphs with the above spectrum exist for infinitely many odd values of μ .

Proposition 2.4 *Let D be a Hadamard 3-design corresponding to a Hadamard matrix of order 4μ and let D' be a subdesign of D consisting of all points and the blocks of 2μ arbitrary parallel classes. Then the incidence graph Γ of D' has the spectrum of a Hadamard graph of degree $k = 2\mu$.*

Proof. Let N be the point-block incidence matrix of D' . Then $N^T N = \mu(J_{2k} - K) + kI_{2k}$, wherein $K = J_2 \otimes I_k$ (\otimes denotes the Kronecker product and the indices indicate the size). So the spectrum of $N^T N$ is $\{(k^2)^1, k^k, 0^{k-1}\}$. If A is the adjacency matrix of Γ , then

$$A = \begin{bmatrix} \mathbf{0} & N \\ N^T & \mathbf{0} \end{bmatrix} \text{ and } A^2 = \begin{bmatrix} NN^T & \mathbf{0} \\ \mathbf{0} & N^T N \end{bmatrix}.$$

Since NN^T and $N^T N$ have the same spectrum, A^2 has spectrum $\{(k^2)^2, k^{2k}, 0^{2k-2}\}$ and hence, because Γ is bipartite, A has spectrum $\{k^1, \sqrt{k}^k, 0^{2k-2}, -\sqrt{k}^k, -k^1\}$. □

Hadamard matrices of order 4μ exist for infinitely many values of μ for odd as well as even μ . In particular there is a Hadamard matrix of order 12, which gives $gr_\Sigma > dr_\Sigma = 0$ for case #14 in the table (in fact we find two such graphs by this construction). Also if μ is even we find non-distance-regular examples. For instance the graph of Hoffman (case 5 in the table) can be obtained in this manner. (There is a unique Hadamard 3-design with 8 points, constituted by the points and planes in $AG(3,2)$. If we delete 3 parallel classes represented by 3 planes through a line we obtain the Hamming 4-cube, but if we delete 3 other parallel classes we get Hoffman's graph.) Graphs constructed above are distance-regular around at least half of the vertices (as follows easily from Lemma 1.2(ii)). This means that for these constructions, transitivity of the automorphism group implies distance-regularity (and therefore is impossible for odd $\mu > 1$). However, in general, having a transitive group and the correct spectrum is not sufficient for distance-regularity. This is illustrated by the following example.

Proposition 2.5 *Let $H'(d, q)$ be the graph defined on the dq^{d-1} lines (q -cliques) of the Hamming graph $H(d, q)$, where two lines are defined to be adjacent if they intersect. Then $H'(d, d)$ has the same spectrum as $H(d, d)$, but is not distance-regular if $d \geq 3$.*

Proof. Let N be the incidence matrix of points (vertices of $H(d, d)$) and lines (d -cliques of $H(d, d)$). Then N is a square matrix, $NN^T - dI$ is the adjacency matrix of $H(d, d)$ and $N^TN - dI$ is the adjacency matrix of $H'(d, d)$. Therefore the two graphs are cospectral. Take $d \geq 3$. Then there are in $H'(d, d)$ two types of pairs of vertices at mutual distance two. They can correspond to parallel lines, in which case they have d common neighbours, or to skew lines, in which case there is just one common neighbour. So $H'(d, d)$ is not distance-regular. \square

In particular we find $gr_\Sigma > dr_\Sigma$ if Σ is the spectrum of the cubic lattice graph $H(3, 3)$ (case 17 of the table). This disproves a conjecture of Cvetcović, Doob and Sachs [4], p.183. For spectrum #5, 7 and 8, non-distance-regular graphs can be obtained by the switching procedure of Lemma 1.4 (see [7] for case 5). But if $d \geq 3$, $H'(d, d)$ cannot be obtained from $H(d, d)$ by switching. Indeed, switching does not change the number of common neighbours of two vertices in one side of the splitting and therefore, if switching would work, any two vertices of $H(d, d)$ at distance 2 must lie in different parts, which is impossible because of the existence of three vertices at mutual distance 2.

As a last construction we mention that in some cases the distance i ($i \geq 2$) graph of a given distance-regular graph is cospectral with a distance-regular graph, but not distance-regular. For example the distance 3 graph of $H(3, 3)$ has the spectrum of $GQ(2, 4)$ minus a spread, but has diameter 2 and therefore is not distance-regular. This shows that $gr_\Sigma > dr_\Sigma$ for case #18.

3. Computer results

Although separate computer programs had to be written to deal with the cases 7, 13, 14, 17, 18 and 20, there were sufficient similarities among them that a separate discussion of these similarities is warranted. In the case of #20, (Taylor graph) however, it became clear that an exhaustive search was out of the question. Nevertheless we determined all graphs of diameter 3 that are cospectral with the Taylor graph on 28 vertices. We leave a discussion of this until the end.

Let A denote the adjacency matrix of a graph Γ on n vertices and let $\nu(A)$ denote the binary integer (of length $= \frac{1}{2}n(n-1)$) obtained by concatenating the rows of the upper triangular part of A . The *standard form* of A , denoted by $st(A)$, is the matrix such that

$$\nu(st(A)) = \max \{ \nu(P^t A P) : P \in S_n \},$$

where S_n is the set of all $n \times n$ permutation matrices. In our computer search for graphs cospectral with distance regular graphs we constructed them by means of their adjacency

matrices which in the main were assumed to be in standard form. (An exception to this was #14 where it was known from the spectrum that the graph was bipartite. See §3.2) A backtracking algorithm was used to construct such a matrix, one row at a time, beginning with the first. Suppose that after r rows of a possible candidate A have been found we have the following partially completed matrix

$$\begin{bmatrix} A_r & N_r \\ N_r^T & \mathbf{0} \end{bmatrix}, \quad (1)$$

where A_r is a principal submatrix of order r . A simple observation is that if (1) can be completed to an adjacency matrix in standard form, then (1) itself must be in standard form. Thus, to avoid going down a path in our computer search that might have been previously traversed, it was important, for small values of r at least, to verify that (1) was already in standard form. If it were, we would then proceed to the construction of the $(r + 1)$ st row of A , but if not, we backtracked in an attempt to determine another possible r th row. This checking was done using a variant of F.C. Bussemaker's procedure GRAPHPERMUTATIONSTANDARD. While this test was very important, there were several other that were performed first. These we now describe.

3.1. Using eigenvalues.

As above, let A denote the adjacency matrix of the required graph Γ (in standard form). In each of the cases we used the eigenvalues of Γ to find constants α, β, γ and δ such that the matrix C defined by

$$C = \alpha A^2 + \beta A + \gamma I + \delta J, \quad (2)$$

was positive semidefinite and had small rank, ρ say. Generally speaking, C itself would have two (or three) distinct eigenvalues, the greater (greatest) of which we denote by θ . Then clearly any principal submatrix of C must have rank at most ρ and have eigenvalues that lie between 0 and θ . Interpreting this in terms of (1) above, we see that, for each r ,

$$C_r := \alpha(A_r^2 + N_r N_r^T) + \beta A_r + \gamma I + \delta J$$

has rank at most ρ and eigenvalues that lie between 0 and θ . Thus, when (1) had been found as a possible candidate for completion to an appropriate adjacency matrix A , the matrix C_r was tested to see if

$$(a) \text{ rank}(C_r) \leq \rho \quad \text{and} \quad (b) \quad 0 \leq \lambda_{\min}(C_r) \leq \lambda_{\max}(C_r) \leq \theta,$$

where $\lambda_{\min}(C_r)$ and $\lambda_{\max}(C_r)$ are the smallest and largest eigenvalue of C_r , respectively. Of course it was only necessary to apply (a) when $r > \rho$, while (b) proved useful for small values of r . In the case (a) the mechanics of the test depended on whether the coefficients α, β, γ and δ were all rational integers (as in #7, #13, #17 and #18), so that C is an integral matrix. When this was so the following simple observation gave rise to a very efficient test that avoided the possibility of integer overflow when calculating the rank:

For any integral matrix B and any prime p , $\text{rank}_p(B) \leq \text{rank}(B)$, where $\text{rank}_p(B)$ denotes the rank of B over \mathbb{Z}_p .

We thus replaced the condition (a) above by (a') $\text{rank}_p(C_r) \leq \rho$, where p was given the (purely arbitrary) value 101. In the remaining two cases #14 and #20, the matrix C took the form

$$C = D + \sqrt{d}E,$$

where D and E are integral matrices and d is a squarefree integer. Here we treated C as a matrix of ordered pairs (D_{ij}, E_{ij}) of integers and applied test (a'). In all cases the p -rank was determined using Gaussian elimination.

To see if (b) was satisfied an iterative procedure was used to determine $\lambda_{\min}(C_r)$ and $\lambda_{\max}(C_r)$. To avoid undue lengthening of the computational time a fixed number $n = 500$ of iterations was taken. If, after n steps, successive iterations differed by more than ϵ , where we took $\epsilon = 10^{-8}$, the procedure was deemed not to converge and (b) was not applied. If, however, the procedure converged to within the limit ϵ chosen, it was decided to check the following weaker form of (b)

$$\lambda_{\min}(C_r) \geq -10^{-3} \quad \text{and} \quad \lambda_{\max}(C_r) \leq \theta + 10^{-3},$$

simply to avoid the possibility of roundoff errors.

For each of the cases 7, 13, 14, 17, 18, the matrix C , rank ρ and eigenvalue θ used corresponding to (2) are given in the following list:

7.	$C = A^2 - 2A - 3I - 3J,$	$\rho = 5,$	$\theta = 12,$
13.	$C = -4A^2 + 28I + 7J,$	$\rho = 7,$	$\theta = 24,$
14.	$C = 4A^2 + 4\sqrt{6}A - (6 + \sqrt{6})J,$	$\rho = 7,$	$\theta = 24(6 - \sqrt{6}),$
17.	$C = A^2 + 3A - 2J,$	$\rho = 6,$	$\theta = 18,$
18.	$C = A^2 - A - 2I - 2J,$	$\rho = 6,$	$\theta = 18.$

In some cases a variant of the argument given in the proof of Proposition 2.1 gave reasonable upper bounds λ_0 and μ_0 to the values of λ and μ (λ and μ denote the number of common neighbours of two adjacent and nonadjacent vertices, respectively). However, in two of the cases, namely #17 and #18, better upper bounds to λ were obtained by looking at the Hoffman polynomial. We illustrate by considering #18. Here it is easily seen that $A^3 + 3A^2 - 6A - 8I = 24J$, and from this it follows that $[A^3]_{ii} = 8$. Since $[A^3]_{ii}$ is the number of closed walks of length 3 from the vertex i and this is just the number of edges in the neighbour graph Γ_i , it follows that any two vertices that are joined can have at most 4 common neighbours. Thus $\lambda \leq 4$ in this case. Moreover, knowing the number of edges in Γ_i provided some information that enabled us to avoid some paths of the search tree and thus further shorten the computational time involved.

3.2. Case 14 – The Hadamard graph on 24 vertices.

Here the spectrum tells us that the graph must be bipartite so we assumed that its adjacency matrix A took the form

$$A = \begin{bmatrix} \mathbf{0} & N \\ N^T & \mathbf{0} \end{bmatrix},$$

where N is a $(0, 1)$ matrix of size 12. The only difference in our computer search for such matrices A between that of §3.1 was that instead of A being in standard form we assumed that N was in standard form. In this case the standard form of a $(0, 1)$ matrix M of size $v \times b$, denoted also by $\text{st}(M)$, is defined as follows. Let $\nu(M)$ (previously defined for symmetric $(0, 1)$ matrices) denote the binary integer obtained from M by concatenating the rows of M . Then $\text{st}(M)$ is that $(0, 1)$ matrix such that

$$\nu(\text{st}(M)) = \max\{\nu(PMQ) : P \in S_v, Q \in S_b\}.$$

It is easy to verify that if M is in standard form then so also is the matrix M_r comprising the first r rows of M . Thus before proceeding to the possible construction of an $(r + 1)$ st row of the matrix A above, we checked to see whether the matrix N_r was in standard form (using another procedure of F.C. Bussemaker).

3.3. Case 20 – Taylor graph on 28 vertices.

Taylor graphs are distance-regular graphs with intersection array $\{k, \mu, 1; 1, \mu, k\}$. If $k = 2\mu + 1$ we will call them of conference type (because they are related to conference matrices). All Taylor graphs in our table (#1, 7 and 20) are of conference type.

Proposition 3.1 *Let Γ be a graph cospectral with the Taylor graph on $2q + 2$ vertices of conference type which has spectrum*

$$\{q^1, -1^q, \sqrt{q}^{(q+1)/2}, -\sqrt{q}^{(q+1)/2}\}.$$

Suppose further that Γ has diameter 3. Then Γ is switching equivalent to a graph Γ' with an adjacency matrix of the form

$$A' = \begin{bmatrix} J_2 - I_2 & \mathbf{0} \\ \mathbf{0} & A_1 \end{bmatrix},$$

where A_1 , of size $2q$ is the adjacency matrix of a regular graph of degree q with spectrum

$$\{q^1, -1^q, \sqrt{q}^{(q-1)/2}, -\sqrt{q}^{(q-1)/2}\}. \quad (3)$$

Proof. Since Γ has two vertices u and v at distance 3, we may assume that the adjacency matrix A of Γ takes the form

$$\begin{bmatrix} 0 & 0 & 1 & \dots & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 & \dots & 1 \\ & 1 & 0 & & & & & \\ & \vdots & \vdots & & A_{1,1} & & & A_{1,2} \\ & 1 & 0 & & & & & \\ & 0 & 1 & & & A_{1,2}^T & & A_{2,2} \\ & \vdots & \vdots & & & & & \\ & 0 & 1 & & & & & \end{bmatrix}.$$

Seidel switching with respect to rows (and columns) numbered 2 to $q+2$ gives the desired form for A' . By Lemma 1.2(ii) Γ is distance-regular around u (and v) and therefore the blocks $A_{i,j}$ have constant row sum equal to $(q-1)/2$. Hence, after switching, A_1 has constant row sum q . Since Γ and Γ' have the same Seidel spectrum it follows that $A - \frac{1}{2}J$ and $A' - \frac{1}{2}J$ have the same spectrum. Because Γ is regular, the spectrum of $A - \frac{1}{2}J$ is known and, since $\text{rank}(J) = 1$, an eigenvalue of $A' - \frac{1}{2}J$ with multiplicity m (say) is for $m > 1$ also an eigenvalue of A' with multiplicity at least $m-1$. This yields that A' has eigenvalues -1 , \sqrt{q} and $-\sqrt{q}$ with multiplicity at least $q-1$, $(q-1)/2$ and $(q-1)/2$, respectively. From the structure of A' we know that A' has eigenvalues 1 and q . So only two eigenvalues a and b (say) are not yet known. From $\text{trace}(A') = 0$ we have $a + b = -2$ and $\text{trace}(A'^2) = 2 + 2q^2$ gives $a^2 + b^2 = 2$. Hence $a = b = -1$. Thus we have found the spectrum of A' and the spectrum of A_1 follows. \square

It is a consequence of the above that in order to construct all graphs on $2q+2$ vertices with diameter 3 and cospectral with a Taylor graph of conference type it is sufficient to adopt the following procedure.

- (i) Find all regular graphs Γ_1 on $2q$ vertices having spectrum given by (3).
- (ii) Construct a new graph Γ' by adjoining two new adjacent vertices to Γ_1 that are nonadjacent to all vertices of Γ_1 .
- (iii) Determine all regular graphs of degree q in the switching class of Γ' .

Remark. Using the methods of §3.1 we constructed *all* graphs cospectral with the Taylor graph of conference type in the cases $q = 5$ and 9 and discovered that they all have diameter 3, but we were unable to prove this for larger values of q . In fact we suspect it to be generally false, since it is false for the Gosset graph, which is a Taylor graph that is not of conference type (see [7]).

Since it rapidly became clear that when $q = 13$ an exhaustive search as outlined in §3.1 was not feasible, we used the above three steps to find all non-isomorphic graphs on 28 vertices cospectral with the Taylor graph and having diameter 3. The methods applied to

step (i) were similar to those described in §3.1 and therefore require no further explanation. However, a brief description of step (iii) might be in order.

Suppose we have a ∓ 1 adjacency matrix C of a graph Γ that has the Seidel spectrum of the Taylor graph. To switch C into the ∓ 1 adjacency matrix of a *regular* graph of degree q requires that we find a diagonal ± 1 matrix D , say, such that DCD has row sums 1. This may be rewritten as $DCD\mathbf{j} = \mathbf{j}$ (\mathbf{j} denotes the all-one vector), or equivalently, $CD\mathbf{j} = D\mathbf{j}$, which means that $D\mathbf{j}$ is an eigenvector for C corresponding to the eigenvalue 1. Conversely, any ± 1 eigenvector X (corresponding to the same eigenvalue) yields a diagonal matrix $D = \text{diag}\{x_1, x_2, \dots, x_{2q}\}$ that switches C into the ∓ 1 adjacency matrix of a regular graph of degree q . Thus the main part of step (iii) is to find *all* ± 1 vectors X such that $(C - I)X = \mathbf{0}$. To compute these vectors we basically followed the procedure used by Paulus [10] and subsequently by Bussemaker, Mathon and Seidel [3]. In total, when $q = 13$ we found 85 non-isomorphic graphs after step (i), these giving rise, by step (ii) to 36 equivalence classes under Seidel switching. Examination of each of these switching classes using the ± 1 eigenvectors X found by the method of step (iii) showed that they all had regular graphs (of degree q) in their switching classes, the total number found being 515. Since it is impractical to list all these graphs we content ourselves by giving only one representative of each of the 36 switching classes together with the number of regular graphs obtained from each. These and the other graphs found in §3.1 and §3.2 are listed in Appendix 2.

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Appendix 1

Table of all feasible spectra Σ for a distance-regular graph on $n \leq 30$ vertices with diameter $d > 2$ and degree $k > 2$ (except for the complete bipartite graphs minus a complete matching). The number gr_{Σ} gives the number of non-isomorphic graphs with spectrum Σ and dr_{Σ} indicates how many of these are distance-regular. The incidence graph of a symmetric $2-(v, k, \lambda)$ design is denoted by $IG(v, k, \lambda)$.

#	n	k	Σ	distance regular name and intersection array	dr_{Σ}	gr_{Σ}
1	12	5	$\{5^1, \sqrt{5}^3, -1^5, -\sqrt{5}^3\}$	<i>Icosahedron graph</i> $\{5, 2, 1; 1, 2, 5\}$	1	1
2	14	3	$\{3^1, \sqrt{2}^6, -\sqrt{2}^6, -3^1\}$	<i>Heawood graph</i> = $IG(7, 3, 1)$ $\{3, 2, 2; 1, 1, 3\}$	1	1
3	14	4	$\{4^1, \sqrt{2}^6, -\sqrt{2}^6, -4^1\}$	$IG(7, 4, 2)$ $\{4, 3, 2; 1, 2, 4\}$	1	1
4	15	4	$\{4^1, 2^5, -1^4, -2^5\}$	<i>Line graph of Petersen graph</i> $\{4, 2, 1; 1, 1, 4\}$	1	1
5	16	4	$\{4^1, 2^4, 0^6, -2^4, -4^1\}$	<i>Hamming 4-cube</i> = $H(4, 2)$ $\{4, 3, 2, 1; 1, 2, 3, 4\}$	1	2
6	18	3	$\{3^1, \sqrt{3}^6, 0^4, -\sqrt{3}^6, -3^1\}$	<i>Pappus graph</i> $\{3, 2, 2, 1; 1, 1, 2, 3\}$	1	1
7	20	9	$\{9^1, 3^5, -1^9, -3^5\}$	<i>Tetrahedral graph</i> $J(6, 3)$ $\{9, 4, 1; 1, 4, 9\}$	1	6
8	20	3	$\{3^1, 2^4, 1^5, -1^5, -2^4, -3^1\}$	<i>Desargues graph</i> $\{3, 2, 2, 1, 1; 1, 1, 2, 2, 3\}$	1	2
9	20	3	$\{3^1, \sqrt{5}^3, 1^5, 0^4, -2^4, -\sqrt{5}^3\}$	<i>Dodecahedron graph</i> $\{3, 2, 1, 1, 1; 1, 1, 1, 2, 3\}$	1	1
10	21	4	$\{4^1, (1 + \sqrt{2})^6, (1 - \sqrt{2})^6, -2^8\}$	<i>Generalized hexagon</i> $GH(2, 1)$ $\{4, 2, 2; 1, 1, 2\}$	1	1
11	22	5	$\{5^1, \sqrt{3}^{11}, -\sqrt{3}^{11}, -5^1\}$	$IG(11, 5, 2)$ $\{5, 4, 3; 1, 2, 5\}$	1	1
12	22	6	$\{6^1, \sqrt{3}^{11}, -\sqrt{3}^{11}, -6^1\}$	$IG(11, 6, 3)$ $\{6, 5, 3; 1, 3, 6\}$	1	1
13	24	7	$\{7^1, \sqrt{7}^8, -1^7, -\sqrt{7}^8\}$	<i>Klein graph</i> $\{7, 4, 1; 1, 2, 7\}$	1	10
14	24	6	$\{6^1, \sqrt{6}^6, 0^{11}, -\sqrt{6}^6, -6^1\}$	<i>Hadamard graph</i> $\{6, 5, 3, 1; 1, 3, 5, 6\}$	0	4
15	26	4	$\{4^1, \sqrt{3}^{12}, -\sqrt{3}^{12}, -4^1\}$	$IG(13, 4, 1)$ $\{4, 3, 3; 1, 1, 4\}$	1	1
16	26	9	$\{9^1, \sqrt{3}^{12}, -\sqrt{3}^{12}, -9^1\}$	$IG(13, 9, 6)$ $\{9, 8, 3; 1, 6, 9\}$	1	1
17	27	6	$\{6^1, 3^6, 0^{12}, -3^8\}$	<i>Cubic lattice graph</i> $H(3, 3)$ $\{6, 4, 2; 1, 2, 3\}$	1	4
18	27	8	$\{8^1, 2^{12}, -1^8, -4^6\}$	$GQ(2, 4)$ minus a spread $\{8, 6, 1; 1, 3, 8\}$	2	13
19	28	3	$\{3^1, 2^8, (-1 + \sqrt{2})^6, -1^7, (-1 - \sqrt{2})^6\}$	<i>Coxeter graph</i> $\{3, 2, 2, 1; 1, 1, 1, 2\}$	1	1
20	28	13	$\{13^1, \sqrt{13}^7, -1^{13}, -\sqrt{13}^7\}$	<i>Taylor graph</i> $\{13, 6, 1; 1, 6, 13\}$	1	≥ 515
21	30	7	$\{7^1, 2^{14}, -2^{14}, -7^1\}$	$IG(15, 7, 3)$ $\{7, 6, 4; 1, 3, 7\}$	4	4
22	30	8	$\{8^1, 2^{14}, -2^{14}, -8^1\}$	$IG(15, 8, 4)$ $\{8, 7, 4; 1, 4, 8\}$	4	4
23	30	3	$\{3^1, 2^9, 0^{11}, -2^9, -3^1\}$	<i>Tutte's 8-cage</i> $\{3, 2, 2, 2; 1, 1, 1, 3\}$	1	1

Appendix 2

In the following pages we list all graphs found in the computer search for graphs cospectral with numbers 7, 13, 14, 17 and 18 of the table. We give, for each graph, two lines of data. On the first is its standard form, while on the second we give the order of its automorphism group and its orbits. The binary integer representing the standard form is padded out with just sufficient zeros to make its length divisible by 3 and then it is written as an octal integer. The graphs of #7 arise from three Seidel switching classes, but in fact they can all be obtained from just one of them by the switching described in Lemma 1.4. In #14 the two graphs arising from the Hadamard matrix of order 12 (as described Proposition 2.2) correspond to numbers 14.1 and 14.4 while the distance regular graphs are identified as numbers 7.1, 13.10, 17.4, 18.12 and 18.13. Furthermore, 17.3 is the graph constructed in Proposition 2.3 and 18.11 is the distance 3 graph of 17.4.

In the case #20 the format is different since it is not feasible to list all the graphs found. As mentioned in the text there are (at least) 36 switching classes that have the same Seidel spectrum as the Taylor graph and each of these possesses regular graphs in its switching class. In the listing of these below, we identify the switching class with the standard form that is the greatest of all the regular graphs in its switching class (as defined in §3.1). The notation $[x, y]$ means that there are x non-isomorphic regular graphs in the switching class with automorphism group of order y . Number 36 is the Taylor graph.

#7

1. 7770003607402314600317032506125146425121522460533245316252631774
1440 (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20)
2. 7770003607403210700633005264273043415322302260471547451246731574
96 (1 2 3 6 7 8 11 12 15 18 19 20)(4 5 9 10 13 14 16 17)
3. 7770003746003053402241621341024317023462362641116661137535066654
32 (1 2 7 8 13 14 16 17)(3 4 5 6 15 18 19 20)(9 10 11 12)
4. 7770003746003053402231421443024257025462561700517232157633126474
16 (1 2 7 8)(3 4 5 6 15 17 19 20)(9 10 11 12)(13 14 16 18)
5. 7770003746003053402231421443024257025462562541036711317633126474
48 (1 2 7 8 9 10 11 12 13 14 16 18)(3 4 5 6 15 17 19 20)
6. 7770003746003053402241611350060365221462562511116661267272513364
12 (1 2 4 6 7 8 13 14 15 16 18 20)(3 5 9 12 17 19)(10 11)

#13

1. 774000016160000111600011230002003602040160034300015122224422060047644112
45031130504642202054
2 (1 13)(2 23)(3 6)(4 18)(5 16)(7 20)(8 12)(9 22)(14 21)
2. 774000016160000111600011034002101604041140020260054322023415100036332051
30601231103040661005
12 (1 5 6 7 10 11 12 14 16 20 22 24)(2 3 4 8 9 13 15 17 18 19 21 23)

3. 774000016160000111600011034002101604040160022220047422442404520107612124
44611342520204104454
3 (1 11 17)(2 4 10)(3 15 22)(5 16 24)(6 13 23)(7 9 20)(8 14 18)(12 19 21)
4. 774000016160000111600011034002101604040160022220047422244410460107612124
44611342460404110254
2 (1 13)(2 23)(3 6)(4 20)(5 15)(7 12)(8 17)(9 22)(14 21)
5. 774000016160000111600011034002001704101060021120443426021414240143331004
55411260032225242006
16 (1 5 9 10 12 13 15 16)(2 3 4 8 14 17 20 21)(6 7 11 18 19 22 23 24)
6. 774000016160000111600011034002001704101060021120443424222450220143331040
53011260434225242006
8 (1 5 12 15)(2 8 14 17)(3 4 20 21)(6 7 18 19)(9 10 13 16)(11 22 23 24)
7. 774000016160000111600011034002001702110060116100006524112451021023325042
12241222135203020126
6 (1 4 5 12 17 19)(2 7 8 14 15 21)(3 16 20)(6 10 18)(9 11 13 22 23 24)
8. 774000016160000111600011034002001702100070232100026225041450240035211467
00212224306225003403
6 (1 4 5 13 15 19)(2 7 8 9 17 18)(3 16 20)(6 10 14 21 22 24)(11 12 23)
9. 774000014170000210700021114004221402104304022500025411016406150141003174
02452025244452222124
42 (1 23 24)(2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22)
10. 774000014170000210700021016004044602022504402501101422222310250121101130
15241005303060563224
336 (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24)

#14

1. 770000000370000034600004630001252000466000274000016000114001240032005402
44144124144160102113
120 (1 2 3 5 6 7 9 10 11 12)(4 8)(13 14 15 16 17 18 19 20 21 22 23 24)
2. 770000000370000034600006430002053000513001074000006000016000220065005501
64033006056615600000
12 (1 5 7 9 10 12)(2 3 4 6 8 11)(13 14 19 20 21 22)(15 23)(16 17 24)
3. 770000000370000036400004270001132000466000534000016000114001240026004602
60150124144160102113
20 (1 2)(3 4 5 6 7 8 9 10 11 12)(13 14 15 16 17)(18 19)(20 21 22 23 24)
4. 770000000370000036400004270002074001123002146000006000014001460017005200
52003074036746300000
120 (1 2 3 4 5 6 7 8 9 10 11 12)(13 14 15 16 17 20 21 22 23 24)(18 19)

#17

1. 7700000001630000000074000001070000040340030003406000070000340000034444000
002235122011440011345102220047444110020204112
288 (1 2 6 7 8 9 12 13 15 16 18 19)(3 4 5 20 21 22 23 24 25)(10 11 14 17 26 27)

2. 770000001434000002230000000170000004340012222001222200004460220050220050
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72 (1 2 3 6 7 9 10 11 12 13 16 19 20 21 24 25 26 27)(4 5 8 14 15 17 18 22 23)
3. 770000001036000000017000046014000014140070003000300140404200202100060030
300032104021040001200053031403012012746324500
1296 (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27)
4. 770000001036000000017000044214000104140044250000425001100300040060440500
101204001400007001200056420504442043502444163
1296 (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27)

#18

1. 776000001407600002041700000001760000070743104600115014264205215250000176
01026510150462075202301012206250202224410120
4 (1 2 21 22)(3 25)(5 10)(6 9 13 14)(7 12 16 18)(8 11 15 17)(19 24)(20 23)(26 27)
2. 776000001407600002041700000001760000070743104600114414264405215250000176
010165102504620752025010142052302024214410210
36 (1 2 3 6 7 8 9 11 12 13 14 15 16 17 18 21 22 25)(4 5 10 19 20 23 24 26 27)
3. 776000001407600002001740001000760020030742140640306014150740023262014056
014512605001252160522311424101130044041206040
2 (1 4)(3 10)(5 23)(6 21)(7 24)(8 22)(9 20)(11 13)(15 26)(16 17)(18 25)(19 27)
4. 776000001407600002001740000020760040030743063000102154604464132046042124
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12 (1 7 11 17 20 21)(2 3 8 9 13 14 18 19 22 24 25 27)(4 5 23 26)(10 15)(12 16)
5. 776000001407600002001740000020760040030743063000102154426064310446101124
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12 (1 7 12 17 20 21)(2 3 8 9 13 14 18 19 22 24 25 27)(4 5 23 26)(10 15)(11 16)
6. 776000001407600002001740000020760040030743063000102154426064310446101124
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16 (1 7 11 12 16 17 20 21)(2 3 4 5 8 9 13 14 18 19 22 23 24 25 26 27)(10 15)
7. 776000001407600000001760000030361471000400435020044342461043411254000426
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4 (1 10)(2 8)(3 4 26 27)(5 9)(6 7 20 24)(11 12 18 22)(13 14)(17 21)(19 23)
8. 776000001407600000001760000030361470210000514060060152461043403350010414
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9. 776000001407600000001760000030361470210000514060060152461043403350000426
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11. 776000001007700000000770041610140004341444620120412503406122251005020320
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1296 (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27)
12. 776000001007700000000770041414140014141446003140063141140360036014202504
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13. 776000001007700000000770041414140014141446003140063141124250052124202504
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324 (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27)

#20

1. 777740000776074001740017606272141021132430321017241551062345150133413674
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13. 777740000776074001707216006441117006441350013561262471033124705254431170
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[24, 1]
14. 777740000776074001707216006405156021130630322036202276116327025530107246
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16. 777740000776074001707216006405156021130630322036202276116327025530107246
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[4, 1]
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18. 777740000776074001707216006405156005430720116232444274166123033740113036
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[2, 1] [8, 2]
19. 777740000776074001707216006405616005460330054570253230436134027240715024
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23. 777740000776074001746017006212147020362540202551615126063554122544136426
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24. 777740000777460001701276006405750020314560403150761240742122117720247650
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[1, 3] [2, 4] [1, 6]
26. 777740000776074001707216006441117015540214310546414326432530464066525146
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[5, 2] [4, 8]
28. 777740000776074001707216006405156005132430103076242274136704225606344530
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[3, 4] [1, 6] [1, 12]
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[3, 1] [3, 2] [1, 3] [2, 6]
33. 777740000776074001707216006405156005430720116232444274166123033446113330
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[2, 1] [8, 2]
34. 777740000777460001701276006405616006223320066132465260162534032431073340
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[2, 2] [3, 4] [2, 8]
35. 777740000777460001701276006405616006242360065170447260166134033140713030
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[1, 4] [1, 8] [2, 24]
36. 777740000770077001434343402232332022525520514652447214545132070651162544
712250165432462062624234566171511263705065433432326777
[1, 2184]

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